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Resource Allocation Optimization from User Preferences and Utility Parameters

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Abstract: In many organizations a budget director and department heads conduct resource allocation through negotiations at a board room table. In this paper, we propose a more thorough resource allocation process that provides a budget director with a broader understanding of her organization and serves as a foundation for negotiations. The process begins with an employee survey to gauge user preferences of individual resources. From these preferences we calculate a perceived utility score for each resource. Finally, using the perceived utility scores and cost we develop a linear program which recommends resource allocation portfolio that maximizes the portfolio's perceived utility. Through model comparison, we determined that our process delivers a portfolio with 14 percent more perceived utility than a portfolio developed with negotiation alone.

Keywords: Resource Allocation, Linear Programming, Utility Parameters, Survey Design.

1. Introduction

Resource allocation optimization is a topic that has been explored since George Dantzig's development of the simplex method in 1947 and has since been widely applied to business, manufacturing, and engineering practices. Resource allocation is ubiquitous in almost every facet of human life, but each distinct application is unique. Varying techniques are used by different organizations to solve their own resource allocation problems, yet sub-optimal solutions are often presented. A variety of factors cause sub-optimal resource allocation. To start, budget directors are generally far removed from the actual resource utilization. Additionally, comparing dissimilar resources based off their importance to the organization is difficult. How can a budget director accurately value different groups of resources, or even resources within a group? Finally, few technical, quantifiable operations are included in some budgeting processes, and budget directors often resort to negotiation for final allocations.

In this paper, we propose a new technique that improves the budgeting process in three ways. First, it helps close the gap between budget directors and resource utilization by attaining employee input on item importance. Secondly, our technique involves a way to compare items based on their perceived utility in a quantitative manner. Lastly, instead of budget directors jumping straight to negotiations, an optimized resource allocation based off the resources' perceived utilities can be used as an initial framework to build negotiations off of.

1.1. Problem Statement

Depending on the organizational context, resource allocation processes can be administered quite differently, but they often build upon the same general foundation. Typically, organizations conduct their resource allocation at quarterly meetings with a budget director and various department heads who each oversee an individual commodity group (Kulkarni, 2020). These commodity groups are constrained by a common budget and must be balanced to optimally contribute to the organizational purpose. To achieve the ideal balance, meeting attendees list items in each commodity group from least to most important. Thereafter, negotiations between department heads commence, each vying for enough budget to fulfill their commodity group's needs.

This budgeting process leaves much room for improvement. First, the negotiation process lacks a rigorous quantitative foundation. Additionally, the department heads, who work at a high level of the organization, often have little input from lower-level employees as to which resources to prioritize. Lastly, being far removed from the use of the resources, department heads struggle to accurately estimate the utility of distinct resources especially since their comparison often involves complex attributes.

1.2. Related Work

Initially motivated by the need to solve complex planning problems for military operations in World War II, linear programming was developed to determine an optimal solution subject to constraints. Specifically, George Dantzig devised the simplex method in 1947, and American industry saw its widespread application afterwards. The method decreases computational costs by only testing solutions where constraints intersect and comparing their output values from the objective function (Dantzig, 1990). We will use the simplex method later in this paper to optimize a resource allocation portfolio based on total perceived utility.

Utility and game theorists have long struggled with similar resource allocation problems. In the 1960s, a mathematician named Thomas Saaty led a research project for the U.S. Department of State which sought to compare the utility of different weapon systems. Disappointed with the project's findings, he later contended (Saaty, 1996):

> "The theories and models of the scientists were often too general and abstract to be adaptable to particular weapon tradeoff needs. It was difficult for those who prepared the U.S. position to include their diverse concerns ... and to come up with practical and sharp answers."

Thereafter, Thomas Saaty sought to readdress the failures of the project by creating a new systematic approach to solve complex comparisons across maldistributed goods. As such, Saaty engineered the now widely practiced Analytical Hierarchy Process (AHP). AHP is founded off three functions – structuring complexity, measurement through ratio scales, and measurement unit synthesis (Forman & Gass, 2001). AHP as a model structures complex trade offs through hierarchical comparisons, namely rank lists. The process then applies the hierarchical factors to derive (rather assign) ratio-scale measures that can be interpreted as ranking priorities (weights); in turn converting each items' importance into common units allowing for a final direct comparison (Forman & Gass, 2001).

Among other large-scale organizations, the US military has employed AHP in diverse situations since its development. For instance, researchers behind the article "Contested Agile Combat Employment, a Site Selection Methodology" applied Saaty's model to create a site selection decision framework that accounted for varying attributes and optimized agile combat employment infrastructure (Moer, Chini, Feng, & Schuldt, 2022). AHP alone was not enough to account for the complexity of the model, but when combined with multi-attribute utility theory, the model complexity was simplified. The model proposed in this paper utilizes ratios in the same fashion as AHP, but computes respondent scaled preferences instead of integer rankings.

In addition to AHP, surveys have long been used to collect data from otherwise unreachable audiences. The Stated Preference Method (SPM) was developed in 1989 by Eric Kroes and uses individual respondents' statements about their preferences to estimate utility functions. SPM requires purpose-designed surveys for their collection of data. Conjoint analysis, trade-off analysis, and the transfer price method are all topics which fall under the SPM umbrella, and are used to optimize solutions in several fields, including transportation, public works, and environmental projects (Kroes & Sheldon, 1988). Kenneth Willis later uses SPM to determine participants' willingness to pay by directly asking them how much they value a certain good. He found that it was a practical way to estimate the value of goods over the alternative of Revealed Preference, which is less broadly scoped (Willis, 2014). We also found that these surveys must be short and simple to encourage participants to respond accurately (Tourangeau, Michael Brick, Lohr, & Li, 2017).

2. Methodology

2.1. Methodology Overview

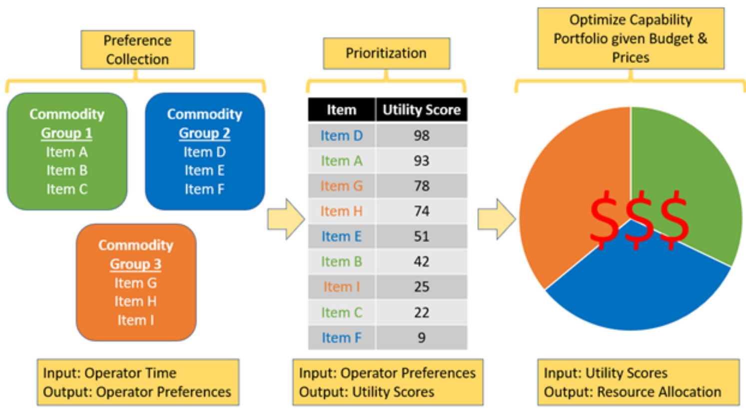

Figure 1: Resource Allocation System Design

To begin, we present an organizational scenario to provide context to our methodology. Say an organization's resources can be assigned to one of three internal commodity groups whose assemblage serves an organizational purpose. Each commodity group is comprised of three distinct items. Each item must be purchased at least once, but no item may be purchased more than five times. The total budget is set at 250 dollars. "Users" are employees who typically work within a commodity group but understand the organizational purpose and how the other commodity groups contribute to it. Essentially, the budget director, who is not part of any individual commodity group, must decide how to allocate funds for resources within each of the commodity groups. Again, she has little insight on the relative utility of a commodity group nor items within it.

Our system is comprised of three main parts that feed data to one another; first the survey (Preference Collection), then the utility parameters (Prioritization), and finally the linear program (Optimized Resource Allocation). To start, a user takes time to complete a survey where they are presented with budgeting decisions similar to ones his budget director will be presented with. The user responses will be compiled to attain an overall user preference for each item. Then, these user preferences will be put through a function to determine perceived utility scores for each item. Please note that we use perceived utility instead of real value since the scores are based off a user's imperfect perception of the item which may be skewed by several factors, including the item's aesthetic (Cascetta & Cascetta, 2009). Finally, a linear program will choose the optimal portfolio using the perceived utility score and price by item along with the budgeting constraints. For a visual representation of our deconstructed system and the perceived utility scores for the items in our organizational scenario, reference Figure 1.

2.2. Survey Design

Using the principles of SPM, we designed a survey that gives users the opportunity to make the budget director's decision for her. For a detailed list of parameters and their definitions, reference the appendix. In the survey, a respondent, $n \epsilon$ N, is presented with an allowance, represented by $a_n \epsilon\ A_N$, and four items randomly selected from the nine available. The items' relative fees, $f_i \epsilon\ F_I$ designate the fee charged for selecting one unit of item i on the survey. The fees are displayed and the respondent must make a decision as to how many of each item he will select. Each selection is represented as $s_{in} \epsilon\ S_{IN}$ where s_{in} designates the amount of item i selected by survey respondent n. Respondents must not exceed their allowances. Additionally, we did not want to include a large number of items in a single survey iteration because of the upper limit on our capacity to process information on simultaneously interacting elements with reliable accuracy and validity (Miller, 1956). A participating user conducts three iterations of the survey (each with a unique combination of items) to maximize data usefulness. It is critical that the survey mimics the resource allocation decisions the budget director faces, while also simple and short enough for users to express their true preferences. Survey respondents can also have their input weighted as more or less important in this model by simply increasing or decreasing their given allowance. For a visual representation of the survey interface, reference Figure 2.

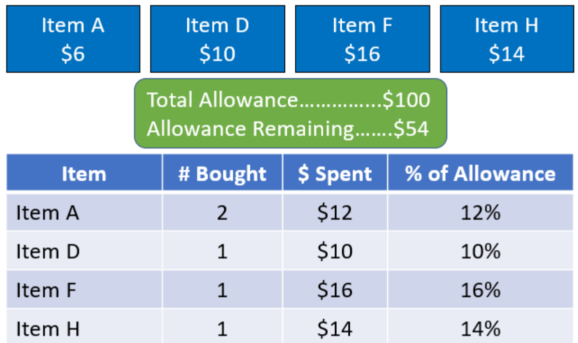

Figure 2: Survey User Interface.

2.3. Optimization Technique

$$\text{maximize} \qquad z = \sum_{i}^{I} u_i \cdot t_i \tag{1}$$

$$\text{subject to} \qquad \sum_{i} p_i \cdot t_i \leq B \tag{2}$$

$$i_{min} \leq t_i \leq i_{max} \tag{3}$$

$$p_i, t_i, u_i \geq 0 \tag{4}$$

The decision variables are $t_i \ \epsilon \ T_I$ where t_i designates the recommended amount of item i to purchase. In line (1) we have our objective function presented in standard form of the simplex method. Let u_i denote the perceived utility of item i, and let z be the total sum of perceived utility. Following the creation of utility parameters, we can use line (1) to maximize perceived utility. Our objective function is subject to budget constraints presented in line (2), where p_i represents the price of item i, and B is an overall set budget. The constraint in line (3) prohibits the quantity of each item selected to buy from exceeding its given maximum and minimum values, while line (4) prohibits the prices, purchase amounts, and utility values to be non-negative. Contracts may commit an organization to purchasing a minimum number of items, but the organization may not want to purchase too many of a specific item. Note how in line (4) the lower bound of t_i is zero. This model will not allow negative numbers of items to be purchased through the use of non-negativity constraints.

2.4. Utility Creation

Equations (5) and (6) are used to create a parameter that represents perceived utility for each item based off the preferences expressed in the survey data. In order to create a common unit of utility, we achieve scaled preferences for items using equation (5) below. Achieving scaled preferences for items has historically been a common approach for solving resource allocation problems, and this first step is loosely based on AHP. However, this model builds upon AHP by including ln $(s_{in} + 1)$. This roughly accounts for diminishing returns and creates comparative scaled preferences for each respondent notated as $r_{in} \ \epsilon \ R_{IN}$ where r_{in} designates the preference for item i for survey respondent n. Scaled preferences have unique trade off ratios, which creates more precise estimators than uniform integer rankings used by Thomas Saaty in AHP (Saaty, 1996).

$$r_{in} = \frac{f_i * s_{in}}{a_n} \cdot \ln{(s_{in} + 1)} \tag{5}$$

Once each item (i) for every respondent n has a correlating r_{in} value, which represents how much respondent n values item i, the average is taken to give each item a perceived utility value. Using sample-average approximations is appropriate in this situation because equation (6) has a structure that enables the efficient optimization algorithm, equation (1), to operate and the limiting function of fees/prices that is actually minimized shares a similar structure such that the local minimums/maximums of fees/prices correlate with the amounts of each item selected by respondents (Kim, Pasupathy, & Henderson, 2015). The known properties of sample-average approximations are proven to converge towards the population average, so as the sample size grows the optimal objective solution provided by (1) also converges towards the true optimal solution (Kim et al., 2015).

$$u_i = \left(\sum_{n}^{N} r_{in} \right) * \frac{1}{N} \tag{6}$$

2.5. Alternative Models

In order to test the performance of our proposed linear programming model, we determine two other resource allocation methods to compare with our model. We cannot reasonably state that our proposed process improves an organization's resource allocation without building a representative model of its current methods. However, before creating a model representative of the negotiation process, we establish a baseline model, which we call the Naive Model. This model serves as a control in our comparison. The Naive Model randomly buys one item from any commodity group until no more items can be bought without going over budget. We acknowledge that any high functioning organization does not conduct their resource allocation in this manner, but the Naive Model serves as a heuristic.

Our second and more realistic model is the Negotiation Model. The Negotiation Model serves to represent a typical resource allocation by way of negotiation at a board room table. We wish to depict department heads going around the table and each choosing an item within their commodity group, presumably with some discussion. In the Negotiation Model, each commodity group first buys the median quantity of their highest perceived utility item. Then, department heads take turns buying the next highest perceived utility items within their commodity group until no more items could be bought without going over total budget. Again, these models are developed as comparisons to our proposed model.

3. Results

The final test comes from running the Naive, Negotiation, and Proposed Models and comparing their recommended resource portfolios. In support of a fair comparison, all three models have the same inputs and constraints. The only difference is in the model's resource allocation algorithm as described in the methodology section. Every item has the same perceived utility score, regardless of the allocation method. As previously stated, the budget for our example scenario is 250 dollars and each item had to be purchased at least once but no more than 5 times. For a summary of the resulting portfolios' cost and total perceived utility, reference Table 1.

Table 1: This table illustrates the success of the proposed model

Model	Portfolio Cost ($)	Total Perceived Utility
Naive	250	1,304
Negotiation	250	1,465
Proposed	250	1,660

Our proposed model delivers 25 and 14 percent more perceived utility than the Naive and Negotiation Models, respectively.

4. Conclusion

Our proposal improves an organization's budgeting process by allowing lower-level employees to be involved in the decision making process, adds unilateral comparisons between items, and provides an optimal resource portfolio through quantitative methods. However, our proposal also has limitations, predominantly in how the model does not include the element

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of diminishing returns. Purchasing the first quantity of an item provides the same perceived utility as the fifth quantity of the same item, which is often not the case in most organizational contexts. Future work should incorporate nonlinear programming techniques that utilize adjacency conditions. If the constraints of the independent variables take on the form of separable non-linear equations, justified linear approximations can be made using piece wise linear equations to account for the properties of constraint curves. This effectively models diminishing returns by making u_i a function of t_i.

There is value in giving a budget director a broader perspective of her organization. By closing the gap between upper management and resource utilization, providing insight on the value of individual resources, and providing a baseline for negotiations, budget directors are empowered to deliver an optimal portfolio that best serves the organizational purpose.

5. Appendix

- $i \epsilon$ I : Let i designate an item within a commodity group.
- $n \epsilon$ N : Let n designate a respondent of the administered survey.
- $a_n \epsilon\ A_N$: Let a_n designate the allowance survey respondent n is presented with.
- $s_{in} \epsilon\ S_{IN}$: Let s_{in} designate the amount of item i selected by survey respondent n.
- $f_i \epsilon\ F_I$: Let f_i designate the fee charged for selecting one unit of item i on the survey.
- $r_i \epsilon\ R_I$: Let r_i designate the average ranking of item i within I across all survey responses.
- $r_{in} \epsilon\ R_{IN}$: Let r_{in} designate the ranking of item i within I for only survey respondent n.
- $p_i \epsilon\ P_I$: Let p_i designate the actual monetary price of item i used by the budget director and simplex algorithm.
- $u_i \epsilon\ U_I$: Let u_i designate the utility value of item i.
- $t_i \epsilon\ T_I$: Let t_i designate the recommended amount of item i to purchase as determined by the simplex algorithm.

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